
Nicholas P. Chotiros and Marcia J. Isakson
Applied Research Laboratories, The University of Texas at Austin, Austin, Texas 78713-8029

(Received 15 February 2008; revised 31 July 2009; accepted 22 December 2009)

The ability of the grain shearing (GS) and viscous grain shearing (VGS) models to relate geophysical and acoustic properties is tested by a method based on the claimed tight coupling between compressional and shear wave speeds and attenuations, which allows the test result to be quantified in a single parameter. The VGS model is claimed to provide a better fit to the measured sound speed and attenuation in sandy sediments below 10 kHz. In situ measurements of wave speeds and attenuations from the Sediment Acoustics Experiment 1999 (SAX99) and published laboratory measurements by Prasad and Meissner [Geophysics 57, 710–719 (1992)] on a number of sand samples were used to test the models. By this metric, the SAX99 data show that the VGS model is no better than the original GS model because the improved agreement of compressional wave speeds at low frequencies is achieved at the expense of gross overestimation of the shear wave attenuation. When applied to the measurements by Prasad and Meissner, it is shown that the GS models are not applicable at any significant confining pressures, and at zero pressure they may only be applicable to a small subset of the sand samples.

© 2010 Acoustical Society of America. [DOI: 10.1121/1.3337240]

PACS number(s): 43.20.Jr, 43.30.Ma [ADP]

Pages: 2095–2098

I. INTRODUCTION

The grain shearing (GS) model\(^1\) may be categorized as a nearly constant \(Q\) (NCQ) model. These models, as a class, have been successfully applied to the modeling of seismic wave propagation.\(^2\) In the GS model, the \(Q\) of the shear wave is actually constant and that of the compressional wave is nearly constant, changing by a factor of 2 over approximately 4 decades of frequency. It is an attractive model because it has a relatively small number of input parameters, but it was unable to track the steep sound speed dispersion observed below 10 kHz measured in the West Florida sand sheet during the Sediment Acoustics Experiments (SAXs) in 1999 and in 2004. At SAX99, there were relatively few measurements\(^1\) below 10 kHz and those data points had been regarded with some skepticism. However, since that time, they have been reinforced by more recent measurements\(^4\) from SAX04 and the trend is undeniable. In the viscous grain shearing\(^3\) (VGS) model, an additional mechanism was introduced to allow the model to accommodate the observed dispersion. The purpose of this letter is to show that the GS and VGS equations may be rearranged to cleanly separate the acoustic and geophysical parameters on opposite sides of the equal sign, and thus test the ability of the GS models to predict acoustic properties from geophysical measurements, and vice versa, using data available in the open literature. By this means, the validity of the GS models may be tested.

II. THEORY

For the GS model, Eq. (6) in Ref. 5 gives a relationship between the complex wave number of the compressional wave and the geophysical parameters. This equation is rewritten below in a form that is applicable to both GS and VGS.

\[
\frac{c_p - j c_o \alpha_p}{\alpha_p} = \left[1 - \chi g(\omega_p) (j \omega_p n T)^s\right]^{-1/2},
\]

where \(c_p\) and \(\alpha_p\) are the speed and attenuation of the compressional wave at frequency \(\omega_p\), \(c_o\) is Wood’s equation sound speed, \(\chi\) is the grain shearing coefficient, \(n\) is the material exponent, and \(T\) is an arbitrary time constant. In Eqs. 45–47 of Ref. 1, \(c_o\) is defined entirely in terms of the geophysical parameters,

\[
\frac{c_o^2}{\rho_o} = \frac{\kappa_o}{\rho_o} = \frac{N}{\kappa_w} N - \frac{1}{\kappa_w},
\]

\[
\rho_o = N \rho_w + (1 - N) \rho_g,
\]
where \( N \) is porosity, \( \kappa_w \), \( \kappa_g \), \( \rho_w \), and \( \rho_g \) are the bulk moduli and densities of the pore fluid and grains, respectively.

The function \( g(\omega_p) \) is the viscous dissipation term, introduced in Eq. (35) of Ref. 5 to convert the GS model into the VGS model. It is rewritten below in terms of a magnitude and a phasor, in which the phase angle \( \phi_p \) is a function of frequency \( \omega_p \) and the viscoelastic time constant \( \tau \). Note that by making \( \tau \) equal to infinity, \( g(\omega_p) \) becomes unity and the VGS model reverts to the GS model.

\[
g(\omega_p) = \left(1 + \frac{1}{j\omega_p \tau} \right)^{-(n-1)/2} e^{i\phi_p}.
\]  

(3)

Following a similar procedure as Eqs. (6)–(11) of Ref. 5, the two sides of Eq. (1) are squared and inverted. Using the identity

\[
(j)^n = \cos\left(\frac{n\pi}{2}\right) + j \sin\left(\frac{n\pi}{2}\right),
\]

(4)

the real and imaginary parts may be separated to give the following equations, which are comparable to Eqs. (10) and (11) of Ref. 5 from GS theory,

\[
\chi(\omega_p \tau)^n = \left(1 + \frac{1}{\omega_p \tau^2} \right)^{(n-1)/2} \cos\left(\phi_p + \frac{n\pi}{2}\right)
\]

\[
= \left(\frac{c_p}{c_w}\right)^2 \left[ 1 - X^2 \right] \left[ 1 + X^2 \right]^2 - 1,
\]

(5)

\[
\chi(\omega_p \tau)^n = \left(1 + \frac{1}{\omega_p \tau^2} \right)^{(n-1)/2} \sin\left(\phi_p + \frac{n\pi}{2}\right)
\]

\[
= \left(\frac{c_p}{c_w}\right)^2 \frac{2X}{\left[ 1 + X^2 \right]^2}.
\]

(6)

where \( X \equiv c_p \alpha_p / \omega_p \).

Dividing one by the other to get a tangent function, the result is an equation that is valid for VGS and GS, and comparable to Eq. (12) of Ref. 5.

\[
\tan\left(\phi_p + \frac{n\pi}{2}\right) = \frac{2X}{\left[ 1 - X^2 \right] - \left(\frac{c_w}{c_p}\right)^2 \left[ 1 + X^2 \right]^2}.
\]

(7)

By a similar process to Eqs. (13)–(15) in Ref. 5, but including a viscous dissipation term, \( g(\omega_s) \), a corresponding tangent identity is obtained for the shear wave.

\[
\tan\left(\phi_s + \frac{n\pi}{2}\right) = \frac{2Y}{1 - Y^2},
\]

(8)

where \( Y \equiv c_s \alpha_s / \omega_s \), \( c_s \) and \( \alpha_s \) are the shear speed and attenuation at frequency \( \omega_s \), and \( \phi_s \) is the phase shift due to VGS viscous dissipation at \( \omega_s \), as defined by

\[
g(\omega_s) = \left(1 + \frac{1}{j\omega_s \tau} \right)^{n-1} = \left(1 + \frac{1}{\omega_s \tau^2} \right)^{(n-1)/2} e^{i\phi_s}.
\]

(9)

Comparing the left-hand sides of Eqs. (7) and (8), it is evident that the two tangent terms would be equal if \( \phi_p \) and \( \phi_s \) were equal. In the case of GS, \( \tau \) is infinite, the values of both \( \phi_p \) and \( \phi_s \) are exactly zero, and equality is always obtained. In the case of VGS, equality may be achieved if the frequencies of the compressional and shear waves, \( \omega_p \) and \( \omega_s \), are equal, since both \( \phi_p \) and \( \phi_s \) are identical functions of frequency, as defined in Eqs. (3) and (9). Equality may be approximately achieved if both \( \omega_p \) and \( \omega_s \) were very much larger or smaller than \( 1/\tau \) because in those cases their values would asymptotically tend toward 0 or \( (1-n)\pi/2 \), respectively. Given equality, the tangent terms may be eliminated from Eqs. (7) and (8), and after some rearrangement, the result is

\[
c^2_w = c_p^2 \frac{X(Y^2 - 1) - Y(X^2 - 1)}{Y(1 + X^2)^2}.
\]

(10)

A similar derivation may be found in Ref. 7. It appears that \( c^2_w \) can be calculated in two different ways: (a) by using the bulk sediment properties via Wood’s equation as defined in Eq. (2) or (b) by using measured wave parameters as defined in Eq. (10). It is proposed that the GS models be tested by comparing the values of \( c^2_w \) obtained by these two different expressions. To avoid confusion, the value defined by Eq. (2) will be called \( c^2_{\text{gw}} \), where the subscript \( w \) denotes that the value depends only on the bulk sediment properties (\( \kappa_w \), \( \kappa_g \), \( \rho_w \), \( \rho_g \), and \( N \)). The value obtained by Eq. (10) will be called \( c^2_{\text{gwv}} \), the subscript \( w \) referring to its dependence only on the wave parameters (\( c_p \), \( X \), \( Y \)).

\[
c^2_{\text{gwv}} = \frac{c_p^2 (1 - X^2)Y - X(1 - Y^2)}{Y(1 + X^2)^2}.
\]

(11)

\[
c^2_{\text{ob}} = \frac{\kappa_w}{\rho_w}.
\]

(12)

To make the comparison simpler, let us define \( R \) as the ratio,

\[
R = \frac{c^2_{\text{gw}}}{c^2_{\text{ob}}}.
\]

(13)

An \( R \) ratio of one supports the GS model, and by making all wave measurements at the same frequency (\( \omega_p = \omega_s \)) the result would also apply to VGS models. The beauty of this method is that the wave properties are cleanly separated from the bulk properties; \( c^2_{\text{gwv}} \) is exclusively a function of the wave properties and \( c^2_{\text{gw}} \) of the bulk properties. The difference between them is a direct indication of the GS models’ ability to connect bulk and wave properties, and this connection may be quantified in terms of just one variable, \( R \).

### III. APPLICATION TO PUBLISHED DATA

At SAX99, the in situ measured shear wave speed and attenuation\(^3\) at 1 kHz were 120 m/s (with a range of 97–147 m/s) and 30 dB/m (with a range of 21–40 dB/m). The in situ compressional wave speed and attenuation measurements were simultaneously made at a number of frequencies between 2 and 200 kHz, in addition to core measurements at 400 kHz, as shown in Figs. 3 and 4 of Ref. 3. These data points are identified in Figs. 1(a) and 1(b). A couple of curves representing the upper and lower bound GS models from the same reference are also shown. From the same reference, the relevant geophysical parameter values for calculating \( c^2_{\text{ob}} \) are \( \kappa_w = 2.395 \text{ GPa}, \ \kappa_g = 32 \text{ GPa}, \ \rho_w \).
and the result is shown in Fig. 2. The range of acceptable properties for the input parameters. The uncertainty in the values of bulk and shear wave frequencies is found by numerical simulation that the acceptable range is computed as a function of the compressional wave frequency \( f \), and assigning a conservative rms error of 0.02 to each of \( c_p \), \( X \), and \( Y \), and a rms error of 0.02 to \( N \), it is found by numerical simulation that the acceptable range of values of \( R \) is within ±0.1 of 1, at the 80% level. Specifically, 80% of the experimental values of \( R \) should lie between 0.9 and 1.1. In Fig. 1(a), it is evident that the measured wave speeds below 10 kHz are not in agreement with the GS model, and in Fig. 2, the corresponding values of \( R \) are less than 0.9. Between 10 and 400 kHz, the \( R \) ratios are within the acceptance zone, indicating that the shear measurements at 1 kHz, in combination with the compressional measurements in this frequency band, are compatible with the GS model, most likely because their behavior in this region is consistent with a nearly constant model. There is a discernible trend in the values of \( R \) and a second order polynomial least-squares fit is superimposed as a plausible trend, and as a way of interpolating its value at 1 kHz, the frequency at which the shear wave parameters were measured. At this frequency, the test is also applicable to VGS, but the interpolated value of \( R \) is far from the acceptable region. This seems to be inconsistent with the purpose of VGS, which is to improve model-data agreement at the lower frequencies as stated in Ref. 5. In fact, there is no inconsistency because VGS achieves improved agreement with compressional wave speed and attenuation at the expense of the shear wave. It is a simple matter to run the VGS model with the parameter values provided in Ref. 5 to find that it predicts a shear wave attenuation coefficient that is an order of magnitude greater than the measured value at 1 kHz, as given on p. 417 of Ref. 3 and with a slope that is no longer consistent with the first power of frequency. When one is intent on fitting a particular set of wave parameters, it is easy to lose sight of the other parameters. The \( R \) ratio test does not favor any particular set of subset of parameters, but it gives a robust indication of the model-data compatibility that includes all the relevant parameters. By this measure, VGS is no better than GS.

To properly test the VGS model, measurements of all four wave parameters at the same frequency are needed. In fact, there are very few published experiments in which all four parameters are measured at the same frequency. The measurements published by Prasad and Meissner are the only ones to our knowledge. They were made at 100 kHz as a function of confining pressure, in both dry air-saturated and water-saturated well-sorted quartz sands. The measurement procedure involves placing the sample, in its natural state, in a cylindrical test cell between two porous pistons, in which compressional and shear wave transducers were embedded. Since the pistons were porous, the pore fluid was not pressurized. The confining pressure between the pistons was increased in discrete steps. After each step, the sample was allowed to settle for 6 hours, to ensure that steady-state had been reached, before taking measurements of porosity, and wave speeds and attenuations. The attenuation was presented in the form of \( 1/Q \). It is easily shown that the \( X \) and \( Y \) parameters in the GS model and the compressional and shear \( Q \) values, \( Q_p \) and \( Q_s \), are related by the following identities:

\[
2X = \frac{1}{Q_p} \quad \text{and} \quad 2Y = \frac{1}{Q_s}.
\]  

(14)

The details of the experimental process and the pitfalls to be avoided are described in Prasad's dissertation. The sand samples are labeled in the form “xM.” The prefix “x” may be either “a” or “p” indicating angular or rounded grains, respectively. The number “M” distinguishes different effective grain sizes. Porosity measurements as a function of confinement pressure, available from Ref. 7, were given for samples p2, p3, p4, and p6. The porosities of the remainder were interpolated from these values according to effective grain size. In fact, the changes in porosity were so small that substantially the same results would have been obtained if an approximate value of porosity, i.e., 0.42, were used. Since these were quartz sand samples, the usual values of density and bulk modulus for quartz sand may be used in the calculation of \( c^2 \): \( \kappa = 36 \) GPa and \( \rho_s = 2650 \) kg/m³. The bulk properties for the water and dry atmospheric air are \( \kappa_w = 2.3 \) GPa, \( \rho_w = 1000 \) kg/m³, and \( \kappa_w = 0.145 \) MPa, \( \rho_w = 1.29 \) kg/m³, respectively. The wave parameters for the calculation of \( c^2 \) are found in Figs. 2–5 of Ref. 6.
For the GS model, it was claimed that the fluid films between asperities are “highly incompressible.” Therefore, the fact that these measurements were made under pressure should be immaterial. The values of the wave parameters measured by Prasad and Meissner showed varying degrees of pressure dependence; therefore, $c_{\text{sw}}$ was calculated as a function of confining pressure, at all pressure samples for which all four wave parameters are available. The computed $R$ ratios for the dry air-saturated sand samples are shown in Fig. 3(a). Contrary to common assumptions, air should be considered as a saturating fluid and its influence is not negligible. Of the three sand samples, the $R$ ratio of sample p6 crosses the acceptance zone at a pressure of approximately 1 MPa. One might say that, for this particular type of sand and at this pressure, the GS models are valid, but this is likely to be just coincidence. With the remaining two sand samples, there is no agreement at all because the $R$ ratio does not come close to the acceptance zone. The fact that most of the $R$ ratio values are negative is very interesting because it is entirely incompatible with the GS models, from which $R$ is defined as the ratio of two positive quantities.

The results for the water-saturated sand samples are shown in Fig. 3(b). Most of the water-saturated samples have positive $R$ ratio values at low confining pressures. The one exception is sample p5: Its grain size is in between those of p3 and p6. Their wave properties are all very similar, except that the compressional wave attenuation of p5 is significantly greater than those of p3 and p6. This suggests that the water-saturated p5 sample may contain a significant concentration of gas bubbles; therefore, p5 will be ignored from this point on. If the $R$ ratio values may be extrapolated to zero pressure, it appears that the curve for a2 may intersect the y-axis within the acceptance zone, while the remainder may do so at values that are significantly lower. This suggests that the GS models may be acceptable at zero or low confinement pressures, and only for some sediments. It is interesting to note that a2 is the only sample with “angular grains.” All the other samples have “rounded grains.” The interpolated $R$ ratio from SAX99 at 1 kHz is also shown in Fig. 3(b). It appears to be consistent with the laboratory data because the curves for p1, p2, p3, and p6 appear to converge on it at zero confinement pressure.

IV. CONCLUSIONS

There are three main conclusions: (1) The GS and VGS equations may be rewritten in a way that puts all the geophysical parameters on one side of the equation sign and all the acoustic parameters on the other. This allows the ability of the GS and VGS models to connect bulk and wave properties to be quantified in just one parameter, the $R$ ratio. (2) The VGS model achieves better agreement with the compressional wave measurements from SAX99, but at the expense of grossly overestimating the shear wave attenuation. It is observed that the GS models are compatible with combinations of shear and compressional wave parameters only within certain frequency bands in which the wave behavior may be approximated as constant $Q$. Outside these frequency bands, particularly in the strongly dispersive band for compressional waves, i.e., between 1 and 10 kHz, both GS and VGS are problematic. (3) When compared to laboratory measurements of compressional and shear waves in sands at 100 kHz by Prasad and Meissner, the GS and VGS models are found to be completely incompatible with wave propagation in air-saturated sand, and possibly compatible with measurements from just one water-saturated sand sample out of the five that were tested, and only when the results are interpolated to zero confinement pressure. It is likely that the proposed tight coupling between compressional and shear wave speeds and attenuations is not valid and different material exponents may be required for shear and compressional waves.

ACKNOWLEDGMENT

This work is sponsored by the Office of Naval Research, Code 321 OA, under the management of Robert Headrick.