A broadband model of sandy ocean sediments: Biot–Stoll with contact squirt flow and shear drag

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Unlike the application of the Biot model for fused glass beads, which was conclusively demonstrated by Berryman [Appl. Phys. Lett. 37(4), 382–384 (1980)] using the experimental measurements by Plona [Appl. Phys. Lett. 36, 259–261 (1980)], the model for unconsolidated water-saturated sand has been more elusive. The difficulty is in the grain to grain contact physics. Unlike the fused glass beads, the connection between the unconsolidated sand grains is not easily modeled. Measurements over a broad range of frequencies show that the sound speed dispersion is significantly greater than that predicted by the Biot–Stoll model with constant coefficients, and the observed sound attenuation does not seem to follow a consistent power law. The sound speed dispersion may be explainable in terms of the Biot plus squirt flow (BISQ) model of Dvorkin and Nur [Geophysics 58(4), 524–533 (1993)]. By using a similar approach that includes grain contact squirt flow and viscous drag (BICSQS), the observed diverse behavior of the attenuation was successfully modeled. © 2004 Acoustical Society of America. [DOI: 10.1121/1.1791715]

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I. INTRODUCTION

For acoustic purposes, the ocean sediment is often modeled as a fluid or an elastic solid. This allows the extensive knowledge and experience in computing elastic wave propagation to be directly applied. The acoustic properties of ocean sediments have been compiled in a number of landmark publications, such as the series of papers by Hamilton in which empirical relationships were developed to connect sound wave speed and attenuation to sediment type and frequency. The data appeared to show that sound speed is approximately independent of frequency, but attenuation increases linearly with frequency. Both vary with sediment type. Methods for inverting acoustic measurements for sediment properties often rely on these relationships. While there has been some success with soft sediments, the results of such inversions in sandy sediments have been poor. The problem is the underlying model.

The inadequacy of the visco-elastic model, in which the sediment is modeled as a visco-elastic solid with approximately constant p- and s-wave speeds and wave attenuations that increase linearly with frequency, has been demonstrated. Model predictions of reflection loss at normal incidence are shown to be inconsistent with measured data. It will be shown below that the visco-elastic model is unable to accommodate the recently measured sound speed dispersion. The poro-elastic model, as conceived by Biot and reformulated by Stoll, has been put forward as a possibly better alternative. The Biot–Stoll model represents the collection of sand grains as a porous elastic frame in which the pore spaces are filled with water. It is certainly an improvement over the homogeneous visco-elastic models but inconsistencies between model and data remained.

At any single frequency, the Biot–Stoll model can be adjusted to match the measured values of acoustic and shear wave speeds and attenuations. When compared to the extensive published experimental data in water-saturated sands, the Biot–Stoll model is unable to match the observed frequency dependence of sound speed and attenuation over a broad range of frequencies. In Sec. II, it will be shown that the model is unable to reproduce the measured sound speed dispersion in the two most comprehensive data sets available to date. It will also be shown that the frequency dependence of attenuation does not follow one consistent power law. It is against this backdrop that the following extension of the Biot–Stoll theory was developed.

In Sec. III, the Biot model with grain contact squirt flow and shear drag (BICSQS) will be developed from simple physical considerations at the grain to grain contact. BICSQS provides a causal and physically sound model for the poro-elastic frame. In Sec. IV, the properties of the BICSQS model will be explored with representative theoretical examples, and matched with the small but growing broadband data base of measured sediment sound speed and attenuation. In the conclusions, the essential points of the BICSQS model are summarized and the remaining issues are described.

II. BROADBAND ACOUSTIC DATA

Acoustic attenuation in ocean sediments, measured as decibels per meter of distance traveled, appears to increase linearly with frequency, as depicted in the data presented by Hamilton (H). Unfortunately, measurements of attenuation in water-saturated sands and sandy sediments are not always consistent with this model. In the interest of fairness to all
Contributing authors and for compactness, published experimental results will be referred to by the first letters of the authors’ names, as follows. Laboratory measurements by Nolle, Hoyer, Mifsud, Runyan, and Ward\(^9\), Thomas and Pace\(^10\), and more recent data from Simpson and Houston\(^11\) (SH), along with in situ measurements by Turgut and Yamamoto\(^12\) (TY) and by several participants at the Sediment Acoustics Experiment of 1999 (SAX99) as reported by Williams, Jackson, Thorsos, Tang, and Schock\(^13\), are overlaid on the historical data from Hamilton in Fig. 1. The trends are confusing. The low frequency data below 5 kHz appear to follow a trend that is closer to \(f^2\). The laboratory data of SH and NHMRW appear to follow a \(f^{-1/2}\) trend. The remainder appear to be linearly proportional to frequency. It would appear that no single power law could satisfactorily match all of the diverse trends.

The sound speed in the sediment is usually assumed to be independent of frequency, but measurements from TY and WJTTTS show quite the opposite. The measurements are reproduced in Fig. 2. Although there are only two such sets of measurements in the published literature, they appear to corroborate each other. Both sets suggest that there may be low and high frequency asymptotic values, and a transition region in the frequency band roughly between 1 and 10 kHz.

It is clear that a simple fluid or elastic model will be incapable of explaining the sound speed dispersion. The recent model by Buckingham\(^14\) (B), which predicts an attenuation that increases as the first power of frequency, also predicts an increase in sound speed with frequency, consistent with Kramers–Krönig. It has five parameters: bulk density, asymptotic low frequency sound speed, frame shear rigidity, frame bulk rigidity, and stress relaxation exponent. The bulk density and the asymptotic low frequency sound speed are measurable. The parameters interact to some extent, but, generally speaking, the frame shear rigidity term controls the shear speed, the average bulk compressive rigidity controls the sound speed, and the stress relaxation exponent controls the magnitude of the attenuation. A realization of this model, using parameter values in Table I, was compared with the data in Figs. 1 and 2. It underestimates the magnitude of the observed sound speed dispersion, and the linear frequency attenuation prediction is unable to match the diversity of frequency dependence observed in the measured data.

The baseline Biot–Stoll model is taken from Stoll and Kan.\(^15\) Two realizations of the baseline model were constructed using model parameter values from the TY and WJTTTS experiments. In the case of TY, the frame moduli were not given, but the shear wave speed and the frame Poisson’s ratio were given. Thus, the frame shear modulus was adjusted to fit the measured shear speed of 110 m/s, and the frame bulk modulus was computed using the Poisson’s ratio of 0.3. The log decrements were set at 0.15 as given in TY for typical cases. The values of permeability \(k\), porosity \(\beta\), added mass coefficient \(c\), and the densities \((\rho_r, \rho_f)\) and bulk moduli \((K_r, K_f)\) of the grain and fluid were given. The pore size parameter \(a\) was computed using the Kozeny–Carman equation.\(^16\)

![Fig. 1. Measured sound attenuation as a function of frequency from several sources and the baseline model predictions.](image1)

![Fig. 2. Measured sound speed as a function of frequency from TY and WJTTTS and baseline model predictions.](image2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_o), low frequency sound speed</td>
<td>m/s</td>
<td>1600</td>
</tr>
<tr>
<td>(n), stress relaxation exponent</td>
<td></td>
<td>0.090 14</td>
</tr>
<tr>
<td>(\gamma_p), compressional rigidity coefficient</td>
<td>GPa</td>
<td>0.248</td>
</tr>
<tr>
<td>(\gamma_s), shear rigidity coefficient</td>
<td>GPa</td>
<td>0.001 52</td>
</tr>
<tr>
<td>(\rho_o), bulk density of saturated medium</td>
<td>kg/m(^3)</td>
<td>2016</td>
</tr>
</tbody>
</table>

**Table I. Parameter values of the Buckingham model.**

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\(c_o\), low frequency sound speed m/s 1600

\(n\), stress relaxation exponent 0.090 14

\(\gamma_p\), compressional rigidity coefficient GPa 0.248

\(\gamma_s\), shear rigidity coefficient GPa 0.001 52

\(\rho_o\), bulk density of saturated medium kg/m\(^3\) 2016
where $k_k$ is the Kozeny parameter, which has a value of 5 for spherical grains. In the case of WJTTS, all of the parameter values were given. The values are shown in Table II. The computed sound speed and attenuation are compared with the data in Figs. 1 and 2. The measured dispersion is clearly greater than the TY baseline Biot–Stoll model. The WJTTS baseline model appears to predict greater dispersion, and comes closer to matching the measured dispersion. However, the near-agreement between model and data was achieved by stretching certain parameter values beyond what is likely to be true. In particular, the value of porosity used, 0.385, was significantly higher than the average measured value\textsuperscript{17} from diver cores of 0.366, which may be rounded up to 0.37. The value of the grain bulk modulus chosen was at the lower limit of the 95% confidence interval,\textsuperscript{18} i.e., 32 GPa instead of the usual\textsuperscript{19} 36 GPa for quartz. With respect to attenuation, both baseline models predict a frequency dependence closer to $f^{1/2}$ over the range of frequencies shown. The baseline model is clearly overwhelmed by the diversity in the experimental trends, which suggests that the underlying physics may be more complicated than the model permits.

### III. THE BIOT–STOLL MODEL WITH CONTACT SQUIRT FLOW AND SHEAR DRAG (BICSQS)

The physics of the grain to grain contact is critical to understanding the dispersion and attenuation curves. The baseline model considers the frame to be a monolithic solid that has been hollowed out by interconnected pores, and the properties of the dry frame remain unchanged when fluid is introduced into the pore spaces. This is not the case with fluid-saturated unconsolidated particles because the fluid, through capillary and other short-range forces, alters the mechanical properties of the contact region. In the case of water and sand, the water tends to permeate the grain to grain contact region. Sand grains may be considered as particles with a level of small-scale surface roughness, as illustrated in Figs. 3(a) and (b). Two adjacent sand grains may make solid contact at a few discrete points. Otherwise, there is a layer of fluid between the surfaces. The geometry may be idealized as two flat surfaces separated by a film of fluid, and punctuated by a small solid connection, as in Fig. 3(c).

As the frame is compressed, the two surfaces are pushed closer together. The force between the surfaces is balanced by the stiffness of the solid connection, and by the reaction of the fluid film. With reference to Fig. 4(a), the response to a compressive force $F$ is treated in terms of the change in gap width $y$, and a change in radial displacement of the gap fluid film $r$. The response to a shear force $S$ is in terms of the deformation angle $\theta$, as illustrated in Fig. 4(b).

The compressive response of the solid connection is modeled as a spring, and the response of the surrounding fluid as a spring and a dash-pot, representing the compressibility of the fluid and the drag associated with squirt flow in the gap, as shown in Fig. 4(c). There is an additional inertial term due to the mass of the water as it moves into and out of the gap, but it will be neglected because it is comparatively small.
respectively. The coefficient $k_c$ and $k_y$ of the fluid film, defined by the coefficients of the fluid and the radial permeability of the gap:

The equation is obtained from the balance between the fluid pressure and the flow velocity and fluid density within the gap, a simple first order differential equation with constant coefficients may be applied to model the reactive force between the two surfaces.

In compressive motion, $F$ must equal the elastic reaction of the solid contact, represented by the coefficient $k_c$, and the bulk reaction of the fluid film, defined by the coefficients $k_y$ and $c_r$ with respect to first order changes in $y$ and $r$, respectively. The coefficient $k_c$ is related to the size and shape of the solid contact and the elastic properties of the solid material, and similarly $k_y$ and $c_r$ are related to the size and shape of the fluid film and its bulk modulus. A second equation is obtained from the balance between the fluid pressure differential and the drag force represented by the drag coefficient $b$, which is determined by the viscosity of the fluid and the radial permeability of the gap:

$$F = (k_c + k_y)(y - y_o) + c_r(r - r_o),$$  \hspace{1cm} (2)

$$b \frac{dr}{dt} = -k_y(y - y_o) - c_r(r - r_o).$$  \hspace{1cm} (3)

All the coefficients are treated as constants, and the time dependence of the solution is assumed to be of the form $e^{-iat}$. The resulting expression for the contact stiffness, i.e., the ratio between applied force and $y$-displacement amplitudes, is obtained in terms of two constants and a relaxation frequency, $\omega_k$. Setting

$$F = A_f e^{-iat},$$  \hspace{1cm} (4)

$$y - y_o = A_y e^{-iat},$$  \hspace{1cm} (5)

the solution for the compressive stiffness of the gap is

$$\frac{A_f}{A_y} = k_c + \frac{k_y}{1 + i \frac{\omega_k}{\omega}}$$

where $\omega_k = \frac{c_r}{b}$.  \hspace{1cm} (6)

It shows that at very low frequencies, the solid contact, represented by the spring, will dominate. At very high frequencies, the fluid has not the time to flow into or out of the gap, and, due to the larger contact area of the fluid film, the compressibility of the fluid film will dominate. Between the extremes, there is a transition region, in which viscous drag will dominate.

On a microscopic scale, the frame is stiffened preferentially along the normal of the grain to grain contact. On a macroscopic scale, in the context of wave propagation and where the wavelength is much greater than the grain size, the aggregate effect of numerous grain to grain contacts within an elemental volume dominates. In this case, the contact orientation may be considered as randomly and homogeneously distributed over angle space, and the resulting effective frame bulk and shear stiffness is isotropic for practical purposes.

This model is practically identical to the Biot plus squirt flow (BISQ) model of Dvorkin and Nur.\textsuperscript{20} However, there are a few differences: Dvorkin and Nur considered squirt flow as being perpendicular to the longitudinal wave direction, but here squirt flow is within the fluid film at the grain to grain contact which may have any orientation. They considered squirt flow to be governed by the same permeability as the longitudinal flow and a characteristic flow length $R$, but, in this case, the flow length is the radius of the contact area and the squirt flow takes place within the grain to grain contact, which is a tightly confined region with a significantly different permeability. Nevertheless, the general characteristics of the models, i.e., the low and high frequency asymptotes and a transition governed by a relaxation frequency, are similar.

The shear response of the grain to grain contact region, controlled by the elastic response of the solid contact and the viscous response of the fluid film, is modeled as a simple spring and dash-pot as shown in Fig. 4(d). The shear deformation angle $\theta$ is related to the shear $S/y_o$ stress by

$$\frac{S}{y_o} = g_c \theta + h \frac{d^2 \theta}{dt^2},$$  \hspace{1cm} (7)

where $g_c$ is the shear stiffness of the solid contact and $h$ the shear drag coefficient associated with the fluid film.

The solution for the net shear stiffness is given by

$$\frac{S}{y_o} = A_y e^{-iat},$$  \hspace{1cm} (8)

$$\theta = A_\theta e^{-iat}.$$  \hspace{1cm} (9)

$$A_y = A_\theta \left(1 - i \frac{\omega}{\omega_k} \right),$$  \hspace{1cm} (10)

where $A_y = \frac{g_c}{h}$.

At very low frequencies, the solid contact will dominate. At very high frequencies, it would appear that the viscous drag will increase monotonically, which is unphysical. In practice, there will likely be a high frequency asymptotic shear stiffness, similar to the compressive stiffness. However, the asymptotic shear stiffness is expected to occur well beyond the highest of frequencies of interest. Therefore, the above
approximation is expected to be adequate for practical purposes.

Finally, the contact model may be related to the effective frame bulk and shear moduli. Since the stiffness of water is more than an order of magnitude less than that of quartz and the other minerals that make up the solid particles, the frame strain must occur mainly at the grain to grain contacts. Given that the grain compressibility and susceptibility to shear are negligible compared to that of the contact, the above results can be scaled up to the effective frame bulk $$K_b$$ and shear $$\mu$$ moduli,

$$K_b = K_c + \frac{K_y}{1 + i \frac{\omega_k}{\omega}}$$,  
$$\mu = G_c \left( 1 - i \frac{\omega}{\omega_h} \right)$$, 

where $$K_c$$ and $$G_c$$ are the asymptotic frame bulk and shear moduli at the low frequency limit, $$K_y$$ is the difference between the asymptotic high and low frequency values of the bulk modulus, and $$\omega_k$$ and $$\omega_h$$ are the bulk and shear relaxation angular frequencies. Let us define the relaxation frequencies, in Hz, as

$$f_k = \frac{\omega_k}{2\pi}$$,  
$$f_\mu = \frac{\omega}{2\pi}$$.

This will be called the Biot–Stoll plus grain contact squirt and shear flow (BICSQS) model in recognition of the prior work by Dvorkin and Nur. The complex frame moduli in the Biot–Stoll model will be computed using the above equations. There is no increase in the number of parameters because the four constants associated with the complex frame moduli, i.e., the real and imaginary parts of the frame bulk and shear moduli, are replaced by computed values using the four independent parameters, i.e., $$G_c$$, $$K_y$$, $$f_k$$ and $$f_\mu$$. The remaining parameter $$K_c$$ is not independent because it is related to $$G_c$$ by the low frequency frame Poisson’s ratio. The frequency dependent frame moduli have two advantages over the constants that they replace: (1) They are computed from a physically sound mechanical model at the grain to grain contact level, and (2) the resulting model is entirely causal. According to the Hertz–Mindlin model, the frame Poisson’s ratio should be less than 0.1, but measurements reported by Bachrach, Dvorkin and Nur21 in dry sand and glass beads indicated a value of 0.15. This is the value that will be used to compute $$K_c$$.

IV. PROPERTIES OF THE BICSQS MODEL

The properties are explored as follows. Let us start with the parameter values of the baseline TY model, and replace the constant frame moduli with the above calculations. The characteristics of the sound speed and attenuation dispersion depend on the values of the contact relaxation frequencies. The following cases will be used for illustration purposes.

A. Case 1: High (infinite) bulk and shear contact relaxation frequencies

If the relaxation frequencies are well above the frequencies of interest, then the frame bulk and shear moduli retain

<table>
<thead>
<tr>
<th>TABLE III. Example cases of the BICSQS model.</th>
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<tbody>
<tr>
<td><strong>Parameter</strong></td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Bulk properties</td>
</tr>
<tr>
<td>$\beta$, porosity</td>
</tr>
<tr>
<td>$\rho_s$, grain density</td>
</tr>
<tr>
<td>$\rho_f$, fluid density</td>
</tr>
<tr>
<td>$K_s$, grain bulk modulus</td>
</tr>
<tr>
<td>$K_f$, fluid bulk modulus</td>
</tr>
<tr>
<td>Fluid motion</td>
</tr>
<tr>
<td>$\eta$, fluid viscosity</td>
</tr>
<tr>
<td>$\kappa$, permeability</td>
</tr>
<tr>
<td>$a$, pore size</td>
</tr>
<tr>
<td>$c$, virtual mass coefficient</td>
</tr>
<tr>
<td>Frame response</td>
</tr>
<tr>
<td>$\mu$, frame shear modulus</td>
</tr>
<tr>
<td>$K_y$, frame bulk mod. difference</td>
</tr>
<tr>
<td>$f_k$, bulk relaxation frequency</td>
</tr>
<tr>
<td>$f_\mu$, shear relaxation frequency</td>
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their low frequency asymptotic values. This is the baseline Biot–Stoll model with constant coefficients. The attenuation goes as $f^2$ at low frequencies and as $f^{1/2}$ at high frequencies.

**B. Case 2: Low bulk and high (infinite) shear contact relaxation frequencies**

The frame bulk modulus will change with frequency, transitioning from its low frequency to its high frequency values in the vicinity of the relaxation frequency, where the imaginary part goes through a peak, as illustrated in Fig. 5(a). The frame shear modulus remains constant and real. Three different values of the bulk relaxation frequencies are illustrated, 3, 30 and 300 kHz. The parameter values are shown in columns 1, 2 and 3 of Table III. The fast, shear and slow wave speeds and attenuation are shown in Figs. 6(a)–(f). The fast wave speed goes through a larger increase in value than the baseline case. The magnitude of the increase is governed by the term $K_y$. The shear speed and attenuation are not significantly affected. The slow wave speed goes through a very significant increase as frequency increases beyond the relaxation frequency. Of course, in a numerical model, it is not possible to use infinity as the shear relaxation frequency, but a large finite value of $10^{10}$ Hz was adequate for practical purposes.

**C. Case 3: High (infinite) bulk and low shear contact relaxation frequencies**

The imaginary part of the shear modulus increases linearly with frequency beyond the relaxation frequency, as illustrated in Fig. 5(b). In this case the sound speed dispersion is small, as in case 1, but the attenuation goes almost as the first power of frequency. Two different values of the shear relaxation frequencies are illustrated, 30 and 300 kHz. The parameter values are shown in columns 4 and 5 of Table III. The fast, shear and slow wave speeds and attenuation are shown in Figs. 7(a)–(f). The shear wave speed increases rapidly beyond the relaxation frequency. There is also a rapid increase in the fast wave attenuation. The shear attenuation is also modified.

Therefore, depending on the values of the contact relaxation frequencies, it is possible to have a variety of frequency dependencies, with exponents ranging from $\frac{1}{2}$ to 2. Recalling the measurements plotted in Fig. 1, it is postulated that...
V. APPLICATION TO EXTANT MEASUREMENT DATABASE

In all cases, the grain bulk modulus was assumed to be that of quartz since quartz sand was used in all of the experimental data. The grain density was assumed to be 2650 kg/m³, except in the case of WJTTS where the grain density was specified as 2690 kg/m³. For the laboratory experiments using fresh water, the fluid density was set to 1000 kg/m³ and the fluid bulk modulus was expected to be in the region of 2.14 GPa, appropriate for fresh water at room temperature and atmospheric pressure.

The BICSQS model was applied to the TY experiment as follows. (1) The model was matched to the low frequency shear wave speed measurement by adjusting the value of $G_c$. (2) The model was matched to the low frequency sound speed by adjusting the fluid bulk modulus. Ideally, this should have happened without any adjustment, because the sound speed lower bound is simply given by the Wood equation for a suspension of the same porosity. However, it was found that the Wood equation sound speed (1585 m/s) was slightly higher than the lowest measured value (1575 m/s). There are a few possible causes. The simplest one is that the value of the pore fluid bulk modulus may be lower than the expected value of 2.3 Pa, due to the presence of microscopic gas bubbles. A slightly reduced value of 2.25 GPa allowed the model to match the low frequency sound speed. This corresponds to a gas volume fraction of only 2.8 parts per million (ppm) as shown in Table IV. No measurements were made of pore fluid gas fraction. (3) The values of $K_y$ and $f_k$ were adjusted to match the measured high frequency sound speed and the transition frequency. (4) The value of $f_m$ was adjusted to match the slope of the measured attenuation curve at high frequencies. In this case, $f_m = \infty$ gave the best fit. (5) The value of permeability was adjusted to match the measured absolute values of attenuation. The best-fit value of permeability was consistent with the measured value reported by TY. The pore size was computed using Eq. (1), and the virtual mass coefficient was provided by TY. The model parameters are shown in column 1 of Table IV and the results are shown in Figs. 8 and 9. The BICSQS model is unable to track the oscillations in the measured curves but it tracks the underlying trends of both the sound speed and attenuation as a function of frequency.

The NHMRW experiment involved four sand samples, but, for brevity, only the results from the samples with the smallest and largest attenuation will be modeled. The experiment did not include a measurement of shear wave speed, therefore, $G_c$ was set to a default value of 0.05 GPa. Measurements of porosity and permeability were provided. Only an average value of the sound speed was given, therefore it was not possible to make a determination of the value of $f_k$, and the same value that was found for TY was assumed.

![FIG. 8. Comparison of sound speed as a function of frequency in the TY experiment and the BICSQS model.](image)
The value of \( K_y \) was adjusted to fit the measured sound speed at 300 kHz—the center of the frequency band. The value of the fluid bulk modulus was not explicitly provided, but it was estimated to be 2.14 GPa, corresponding to distilled water at room temperature. A slightly lower value of 2.10 GPa was required to fit the measurements. The added mass term was computed using Eq. (13) from Ref. 22.

\[
c = 1 + r \frac{(1 - \beta)}{\beta},
\]

where the value of \( r \), a dimensionless constant, was inverted from the measured values of \( c \) and \( \beta \) from the TY experiment (\( r = 0.1885 \)). To fit the slope of the attenuation curve, again, \( f_\mu = \infty \) gave the best fit. To fit the measured absolute values of attenuation, the value of permeability was adjusted. The values of permeability that gave the best fit to the highest and lowest measured attenuation curves, 76 and 0.7 \( \mu \)m\(^2\), are somewhat smaller than the corresponding measured values, 242 and 6 \( \mu \)m\(^2\), reported by NHMRW using a constant flow method. It is likely that the permeability for acoustic motion may be significantly different than that of a constant flow. It is also possible that the samples used in the permeability measurement were not as well compacted as the samples used in the acoustic measurements, giving rise to the higher measured values. The pore size parameter was calculated according to Eq. (1) with \( k_k = 5 \), and the added mass fraction was computed using Eq. (15). The model parameters are shown in column 3 of Table IV and the results are shown in Fig. 11.

The SH experiment contained a measurement of sound speed as a function of frequency but it was not adequately precise to show any deviation from the mean value of 1680 m/s. As in the previous cases, there was no measurement of shear wave speed and the same default values of \( G_c \) were the same default values of \( G_c \) and \( f_k \) are used. The value of \( K_y \) was adjusted to fit the measured sound speeds. To fit the measured slopes of the attenuation curve, the required values of \( f_\mu \) were found to be 400 and 320 kHz. The permeability was adjusted to fit the absolute measured values of the attenuation. No measurements of permeability were provided by TP. The pore size parameter was calculated according to Eq. (1) with \( k_k = 5 \), and the added mass fraction was computed using Eq. (15). The model parameters are shown in column 3 of Table IV and the results are shown in Fig. 11.

The TP experiment involved five sand samples but only the largest and smallest attenuation cases will be modeled for brevity. As in the previous data set, there was no measurement of shear wave speed and only one average sound speed measurement was given for each sand sample, therefore, the

FIG. 9. Comparison of attenuation as a function of frequency in the TY experiment and the BICSQS model.

FIG. 10. Sound speed and attenuation as a function of frequency in the NHMRW experiment and the BICSQS model.

FIG. 11. Sound speed and attenuation as a function of frequency in the TP experiment and the BICSQS model.
used. The frequency range of the attenuation measurement was adequately extensive to be sensitive to the value of $f_k$ and $K_y$, particularly at the low end of the band. Their values were adjusted to obtain the best fit to the shape of the curve. To fit the attenuation curve at the high end of the band, the value of $f_m$ was adjusted. To fit the measured mean value of sound speed, it was necessary to reduce the fluid bulk modulus, from the given value of 2.2 to 1.95 GPa, corresponding to a gas volume fraction of 11.3 ppm. Finally, the value of permeability was adjusted to fit the absolute value of attenuation. The best fit value, $28 \, \mu m^2$, was reasonably close to the measured value of $39.8 \, \mu m^2$. The parameter values are shown in column 4 of Table IV, and the results are in Fig. 12.

Finally, the WJTTS experiment was modeled using the same procedure as for the TY experiment. In modeling the WJTTS experiment, the grain bulk modulus was set at the value for quartz of 36 GPa, rather than the somewhat lower value adopted by WJTTS, and the porosity was set at 0.37, the value found from core sample measurements, rather than the higher value adopted by WJTTS. To match the low-frequency sound speed, again, it was necessary to reduce the given value of fluid bulk modulus, from 2.395 to 2.15 GPa, equivalent to a gas fraction of 35.4 ppm. Attempts were made to measure the pore water gas fraction, but, due to the sensitivity limits of the equipment, it could only be determined that the gas fraction did not exceed 150 ppm. The values of $K_y$ and $f_k$ that best fit the sound speed curve are shown in column 5 of Table IV. The value of $f_m$ was adjusted to match the shape of the attenuation curve. The resulting value, 56 kHz, is the lowest of the five experimental data sets. Finally, to match the absolute values of the measured attenuation the required value of permeability was $115 \, \mu m^2$, which is higher than the measured value of $25 \, \mu m^2$ using the constant flow method. The results are shown in Figs. 13 and 14.

VI. CONCLUSIONS AND DISCUSSIONS

In the BICSQS extension of the Biot–Stoll model, a new representation of the elastic response of the frame is proposed. In the baseline Biot–Stoll model, the frame response is described in terms of complex bulk and shear moduli. Since they are impossible to measure, they were treated as free parameters, and they were adjusted to fit measured values of the compressive and shear wave speeds and attenuations. The values were assumed to be constant. As a result, the resulting model was not causal, and values obtained at one frequency often did not apply at other frequencies.

The extension employs a physical model of the grain to grain contact, which includes squirt flow and shear drag, to compute the frame moduli. The contact is modeled as a pair of relaxation processes in compression and shear. The model
is causal and based on the physics of fluid flow, and applicable over a broad range of frequencies. It differs from the commonly used approach of complex frame moduli with constant coefficients, which is noncausal, and has no physical analog. There is no net increase in the number of model parameters, since the four constants of the frame moduli are replaced by the four parameters of the new frame model.

The model is capable of matching the sound speed dispersion and the diverse frequency dependencies that are found in the extant measurement database. This was demonstrated with the five most comprehensive experimental data sets in the extant published database.

The four parameters of the BICSQS frame model include the low frequency frame shear modulus, the difference between high and low frequency asymptotic frame bulk modulus values, and the compressive and shear relaxation frequencies. The low frequency frame bulk modulus is computed from the low frequency shear modulus via the low frequency Poisson’s ratio. Although the Poisson’s ratio is not directly measurable, it is expected to be equal to the Poisson’s ratio of the dry frame, which has been measured for several granular media.

The frame shear and bulk moduli are coupled via the dry frame Poisson’s ratio, but only in the low frequency limit. At higher frequencies, the fluid film at the grain to grain contact preferentially stiffens the frame bulk modulus and effectively decouples the bulk and shear moduli.

The difference between high and low frequency asymptotic frame bulk modulus values represents the stiffening of the grain to grain contact by the fluid film. The frame bulk stiffness radically changes at the bulk relaxation frequency. The best-fit value of the frame bulk relaxation frequency was found to be in the region from 3 to 5 kHz for all five experimental data sets. The significance of this range of values has not been explored yet, but it is expected to be connected to the average dimensions of the fluid film.

The frame shear relaxation frequency varied from a high of infinity for very clean well-sorted laboratory sands, as in NHMRW, to a low of 56 kHz for \textit{in situ} poorly sorted sand which contained a broad range of grain sizes as well as biological material as in WJTTS. The poorly sorted, but otherwise clean, sand of TP and SH had values in the region of 300–400 kHz. Clearly, a connection between the degree of sorting and the frame shear relaxation frequency is indicated.

There is some argument over whether it is appropriate to use the Biot–Stoll model to explain sound attenuation in water-saturated sand at all in the range of frequencies considered here. The excess attenuation, above the baseline Biot model, has been ascribed to scattering by WJTTS. There are two general types of scatterers: smaller and larger than the acoustic wavelength. If the attenuation is mainly due to small scatterers, such as the sand grains, it would be very significant because scattering by small scatterers is omni-directional. If this were the dominant cause of attenuation, it should increase with grain size. However, the data from TP and NHMRW clearly show that the opposite is true. The measured attenuation decreases with increasing grain size. Therefore, although there are numerous small scatterers, they are not the dominant cause of attenuation in the range of frequencies considered. Scatterers larger than the wavelength may be characterized as volume inhomogeneities, such as spatial variations in sound speed with position. They are weak scatterers that mainly produce forward scattering. They have the effect of distorting the acoustic wave front and destroying coherence without significant loss of total acoustic energy. Undoubtedly, the latter type of scattering occurs in water-saturated sand, but it cannot be the principle cause of attenuation.

Rather than looking to scattering, one should look to permeability. It represents the susceptibility of the frame to pore fluid flow. Using clean graded laboratory sand, it was shown in NHMRW that there is a linear relationship between the permeability and attenuation. The loss mechanism is not scattering but viscous dissipation. The BICSQS model adds two more viscous absorption mechanisms to the baseline Biot model: the squirt flow and the shear drag losses. They are negligible at low frequencies. At intermediate frequencies, the squirt flow losses will dominate, peaking in the vicinity of the bulk relaxation frequency. At higher frequencies, the shear drag losses will kick in and cause the attenuation to increase more steeply with frequency than in the baseline model.

The value of permeability directly controls the absolute value of attenuation. Higher values of permeability correspond to lower values of attenuation. Larger grain sizes produce larger pore spaces, hence higher values of permeability. This explains the observed reduction in attenuation with increasing grain size within the frequency range in which these processes are dominant.

In this study, the value of permeability was adjusted to fit the offset of the measured curve of attenuation versus frequency, and checked against independently measured values where available. Permeability measurements were provided in four of the five data sets considered here. In NHMRW, it was necessary to use a value that was lower than the measured value. In WJTTS it was necessary to use values that were higher than the measured values. In TY and SH, the measured values were consistent with the model values. It appears that measured values of permeability can come within an order of magnitude of the model values. This is consistent with observations that permeability measurements may only be accurate to within an order of magnitude. It is also recognized that permeability measured with constant flow methods may not be a good indicator of permeability for acoustics which involves an oscillating flow, but that is an issue that is beyond the scope of this study.

The low frequency sound speed remains an open issue. Given the nominal bulk properties of the sediment material, it was not possible to explain the measured low frequency sound speed. In this study, a choice was made to assume that the pore fluid bulk modulus was reduced, possibly by the presence of minute concentrations of gas bubbles. The concentrations indicated are up to 40 ppm by volume, which is extremely difficult to measure. Of the five experiments considered, only the SAX99 experiment included attempts to measure the pore fluid gas content, but the apparatus could only resolve 150 ppm or more, therefore a gas fraction of 40 ppm cannot be ruled out. Alternatively, there may be an, as
yet, unknown mechanism or process that depresses the low frequency sound speed. This remains an open issue for future research.

In summary, the procedure for determining the value of permeability and the four BICSQS parameters consists of the following iterative steps, given that all the other Biot parameters are known. (1) The value of $G_c$ was adjusted to match the asymptotic low frequency shear wave speed. (2) The values of $K_y$ and $f_k$ were simultaneously adjusted to maximize the agreement between the model and the measured sound speed dispersion curve. (3) The value of $f_k$ was adjusted to match the slope of the measured attenuation curve at high frequencies. (4) The value of permeability was adjusted to match the absolute measured values of attenuation at high frequencies. Steps (1)–(4) were repeated iteratively until a stable solution was achieved. The terms “low” and “high” frequencies are referenced to the bulk wave relaxation frequency $f_k$, which was estimated to be in the region of 3 kHz for all the cases considered here.

Finally, no attempt was made in this study to compare the model predictions of broadband reflection loss to the measured values. This is expected to lead to further model refinements, particularly the introduction of the “composite medium” submodel.24 This will be pursued in a future study.

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APPENDIX: INSIGNIFICANCE OF INERTIAL TERM AND POISEUILLE FLOW IN THE GRAIN CONTACT SQUIRT FLOW

With respect to the grain contact squirt flow, let us model the grain to grain contact as a fluid film sandwiched between two planar boundaries. The starting point is the expression for the drag force on the fluid film between two parallel planar surfaces given by Biot in Eq. (2.18) of Ref. 3. Using Biot’s nomenclature, and assuming that all quantities are sinusoidal functions of time with a factor $e^{i\omega t}$, the viscous drag force per unit area of the fluid film is given as

$$2\tau = 6\mu U_{1(\omega)} F_1(\kappa_1)/\alpha_1,$$  \hspace{1cm} (A1)

where $\tau$ is the drag force on each wall, $\mu$ is the viscosity, $U_{1(\omega)}$ is the average flow velocity of the fluid film, $\alpha_1$ is half the film thickness, and $F_1(\kappa_1)$ is the frequency dependent correction representing the deviation from Poiseuille friction at high frequencies. The dimensionless term $\kappa_1$ is defined as

$$\kappa_1 = a_1(\alpha_1/\mu)^{1/2},$$ \hspace{1cm} (A2)

where $\mu$ is the fluid density.

At low frequencies, $F_1(\kappa_1)$ tends to 1, and at high frequencies both its real and imaginary parts are approximately equal to 0.234$\kappa_1$. Thus, the magnitude may be approximated by

$$|F_1(\kappa_1)| = (1 + 0.110\kappa_1^2)^{1/2}. \hspace{1cm} (A3)$$

The inertial reaction $R_i$ on the fluid film is simply given by the product of the mass $2a_1\mu$ and the acceleration $i\omega U_{1(\omega)}$

$$R_i = 2a_1\mu |\omega| U_{1(\omega)}. \hspace{1cm} (A4)$$

Taking the ratio of the magnitudes, the approximate result is

$$2|\tau/|R_i| = 3(\mu/\rho) a_1^{-2} a^{-1}(1 + 0.110a_1^2(\alpha_1/\mu)^{1/2}). \hspace{1cm} (A5)$$

For water, the tabulated values of $\rho$ and $\mu$ are 1000 kg/m$^3$ and 0.001 kg/m·s, respectively. For a gap size of 10 $\mu$m or less, and for frequencies less than 100 kHz, the ratio is much greater than 1, indicating that the inertial reaction is insignificant compared to the viscous drag. Furthermore, the value of $|F_1(\kappa_1)|$ remains close to its low frequency asymptotic value, indicating little deviation from Poiseuille flow.

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