A comparison of sediment reflection coefficient measurements to elastic and poro-elastic models

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(Received 15 September 2005; revised 13 May 2006; accepted 2 August 2006)

This work compared three plane wave reflection coefficient models to laboratory sediment reflection measurements collected using a spherical source and receiver. Plane wave decomposition was used to modify the reflection coefficient models to include the inherent spherical effects such as lateral wave interference for direct comparison with the data. Sets of data at two transducer separation distances (1.27 and 0.25 m) were collected to compare the range dependence of the spherical effects. Bandpass filtered linear frequency-modulated chirps from 30 to 160 kHz were used to measure frequency dependence. Grazing angles from 5° to 75° were measured to compare angle dependence. Each set of data was collected along approximately 3 m of smoothed sediment for spatial averaging. Unavoidable experimental effects including transducer response, beam pattern, and spherical spreading were accounted for in order to compare the reflection coefficient measurements with the modified models. Significant spherical wave effects were measured in the data. Three reflection coefficient models were considered: the viscoelastic model, the grain shearing model \cite{Buckingham2000, Williams2001}, and the effective density fluid model \cite{Buckingham2000, Williams2001}. The viscoelastic and grain shearing models predicted values for the reflection coefficient that were not within the 95% confidence interval for low frequencies. The data exhibited high variance which was frequency and angle dependent. This variance is not likely to be caused by variations in bulk properties as defined by the fluid or viscoelastic models. The cause of the variance will be considered in subsequent publications. © 2006 Acoustical Society of America.

[DOI: 10.1121/1.2354002]

PACS number(s): 43.20.Gp, 43.30.Ma [RAS] Pages: 2437–2449

I. INTRODUCTION

There is an increasing need in ocean acoustics to fully understand acoustic wave interaction with sediments. As more and more applications are focusing on shallow water environments, the importance of accurate sediment models is ever increasing. The plane wave reflection coefficient is particularly important in underwater acoustics. Once the reflection coefficient is known, realistic propagation modeling can assist in communication, detection, and classification in littoral environments. Several plane wave models exist that are used to calculate the reflection coefficient given the sediment material parameters. These plane wave models are effective in the majority of situations encountered in real-world activities.

Spherical wave sources present a unique challenge as the plane wave models cannot properly account for the effects of a very broad beam pattern. Using spherical sources in long range applications is often a nonissue since a spherical wave eventually becomes locally planar. Close range applications, on the other hand, are affected greatly by the spherical wave effects. This is particularly important in a laboratory environment, where longer ranges are not feasible.

Measuring acoustic observables in a laboratory environment has many benefits, though. Laboratory measurements can be used to discern which models are most accurate; and assuming an accurate model, they can be inverted to determine well characterized sediment properties. Ideally, the laboratory is a carefully controlled environment where experimental parameters like incident angle, surface roughness and location are all precisely regulated. \textit{In situ} measurements have increased variability that makes comparisons to theoretical models often difficult. In order to compare laboratory measurements made with spherical sources to the plane wave models, the models must first be modified to include the spherical wave effects.

It is a misnomer to call measurements of the bistatic response at the specular angle reflection coefficient measurements when the data are modified by spherical wave effects. However, these measurements will be called, “reflection coefficient measurements” throughout the text for clarity. When spherical wave effects are included in the reflection coefficient, the nomenclature, “spherical wave reflection coefficient” will be explicitly used.

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The work presented here compares experimental spherical wave reflection coefficient measurements to three common plane wave reflection coefficient models that have been modified to include the spherical wave effects. A wide range of frequencies and grazing angles (30–160 kHz, 5°–75°) were tested at two ranges (1.27 and 0.25 m) to accurately compare experimental measurements to predicted model values. The three models that are compared are the viscoelastic model,\(^1\) Buckingham’s grain shearing model,\(^2\) and Williams effective density fluid model (EDFM).\(^3\) The elastic model assumes the sediment has a homogeneous composition and the averaged or bulk parameters for the wet sediment are used. The grain shearing and effective density models are poroelastic models that assume the individual sediment grains interact separately from the surrounding fluid.

The three models were used in plane wave decomposition (PWD) analysis\(^1\) to include the spherical wave effects. PWD represents the spherical wave as a summation of plane waves which are individually modified by the respective plane wave reflection coefficients. The resulting summation which inherently takes into account all spherical wave effects can be directly compared to the measured data. The models and the PWD method will be further discussed in Sec. II.

The experiments were conducted during the summer of 2004 at the Applied Research Laboratories, The University of Texas at Austin. The experimental setup is discussed in Sec. III. Data analysis procedures are presented in Sec. IV. Experimental effects are discussed in Sec. V and results including the comparison of the data to models are shown in Sec. VI. Finally, in Sec. VI, conclusions are drawn from the data/model comparisons.

II. THEORY

A. Viscoelastic model

The viscoelastic reflection coefficient model is a function of the incident angle and the characteristic acoustic impedances of the two mediums. The model includes an additional impedance for the shear wave motion. For the viscoelastic case\(^4\)

\[
R = \frac{Z_{tot} - Z_t}{Z_{tot} + Z_t},
\]

where

\[
Z_1 = \frac{\rho_1 c_1}{\cos \theta_i},
\]

\[
Z_{tot} = Z_c \cos^2 2 \gamma_s + Z_s \sin^2 2 \gamma_s,
\]

\[
Z_c = \frac{\rho_2 \omega}{(k_c^2 - k_s^2 \sin^2 \theta_i)^{1/2}},
\]

\[
Z_s = \frac{\rho_3 \omega}{(k_s^2 - k_s^2 \sin^2 \theta_i)^{1/2}}.
\]

In the above equations, \(R\) is the reflection coefficient and \(Z_t\) is the impedance of the water column. \(Z_{tot}\) is the total impedance of the sediment and \(Z_c\) is the impedance of the compressional wave while \(k_c\) is the complex compressional wave number. \(Z_s\) is the impedance of the shear wave while \(k_s\) is the complex shear wave number and \(\omega\) is the radial frequency. The viscoelastic model is completely defined by seven parameters and the incident angle, \(\theta_i\); the sound speed and density of the water column, \(c_1\) and \(\rho_1\), the sound speed of the compressional and shear waves, \(c_p\) and \(c_s\), the density of the second medium, \(\rho_2\), and the compressional and shear attenuations. The transmission angle of the shear wave, \(\gamma_s\), can be determined using Snell’s law.

Equation (1) predicts a critical angle based on the ratio of the sound speeds both compressional and shear. However, if the shear wave speed in the sediment is much lower than the sound speed in the upper fluid, there will not be a second critical angle. Additionally, the \(Z_t\) term would then be negligible compared to the other impedances. In this case, the elastic reflection coefficient reduces to the fluid reflection coefficient.

B. Effective density fluid model

The sediment can also be considered as a poroelastic medium in which the displacement of the fluid and the displacement of the sand grains are considered independently. In a poroelastic model, there are three possible waves: a compressional wave in which the fluid and sand grain frame are moving in phase with each other, a compressional wave in which the fluid and sand grain frame are out of phase and a shear wave. The out-of-phase compressional wave is referred to colloquially as the “slow” wave. One formulation of a poro-elastic model is the Biot/Stoll model.\(^5\)\(^7\) The Biot/Stoll model has 13 independent parameters, several of which are difficult to measure directly. The parameters are classed into three groups: bulk parameters, fluid motion parameters, and frame parameters. The bulk parameters are porosity \(\beta\), pore size parameter \(\alpha\), grain density \(\rho_g\), fluid density \(\rho_f\), grain bulk modulus \(K_g\), and fluid bulk modulus \(K_f\). The fluid motion parameters include viscosity \(\eta\), permeability \(k\) and tortuosity \(\tau\). Last, in Biot/Stoll theory the sand grains are assumed to form a frame with its own complex bulk and shear moduli. The frame moduli are very difficult to measure, and they are generally assumed to be much smaller than the bulk moduli. If the frame parameters are assumed to zero, the Biot/Stoll equations can be reformulated into fluid-like equations.\(^3\) The poroelastic effects are contained in the “effective density” term and a complex sound speed. The effective density is given by

\[
\rho_{eff} = \rho_f \left( \frac{\alpha(1 - \beta)\rho_g + \beta(\alpha - 1)\rho_f + i\beta F \eta}{\rho_f \omega k} \right)
\]

where \(\rho\) is the sediment density given by \(\beta \rho_f + (1 - \beta)\rho_g\) and \(F\) is a function derived by Biot to account for the deviation from Poissuille flow at higher frequencies.\(^7\) The complex sound speed is represented by
\[
c = \sqrt{\frac{1 - \beta}{K_g} + \frac{\beta}{K_f}}.
\]

The resulting complex density from Eq. (6), compressional sound speed from Eq. (7) and attenuation which given by the imaginary part of \(\omega/c\) from Eq. (7) are used in the fluid model described above.

C. Grain shearing model

Recently, Buckingham has suggested that acoustic reflection phenomena can also be modeled by considering the physics of the grain contact.\(^8\) Translational and rotational shearing of the individual grains against one another amounts to a “random stick-slip” motion statistically combined to produce a relaxation mechanism. Using the average density, the grain shearing model yields the shear and compressional sound speeds and the shear and compressional attenuations, which are input into the elastic model detailed above. According to Buckingham, the equation for the bulk density of the saturated sediment is based on the porosity \(\beta\) and fluid and grain densities \(\rho_f\), \(\rho_g\):

\[
p_2 = \beta p_f + (1 - \beta) p_g.
\]

The equations for the compressional and shear sound speeds \(c_p\), \(c_s\) and attenuations \(\alpha_p\), \(\alpha_s\) are given by

\[
\frac{1}{c_p} = \frac{1}{c_0} \text{Re} \left(1 + \frac{3\gamma_p + 4\gamma_s}{3p_c^0} (i\omega)^n\right)^{-1/2},
\]

\[
\alpha_p = -\frac{\omega}{c_0} \text{Im} \left[1 + \frac{3\gamma_p + 4\gamma_s}{3p_c^0} (i\omega)^n\right]^{-1/2},
\]

\[
\frac{1}{c_s} = \frac{1}{c_0} \sqrt{\frac{p_c^0}{\gamma_s}} |\omega|^{-n/2} \cos \left(\frac{n\pi}{4}\right), \quad \text{and}
\]

\[
\alpha_s = \frac{1}{c_0} \sqrt{\frac{p_c^0}{\gamma_s}} |\omega|^{-n/2} \sin \left(\frac{n\pi}{4}\right).
\]

Here \(\rho\) is the sediment density defined in Eq. (8). The compressional wave speed in an equivalent suspension, \(c_0\), is determined by Wood’s equations.\(^8\) The radial stress-relaxation exponent, \(n\), is computed from initial empirical measurements. The compressional and shear rigidity coefficients, \(\gamma_p\) and \(\gamma_s\), are solved simultaneously from empirical measurements as well.

D. Plane wave decomposition

Since sediments have an angular dependent reflection coefficient which varies in amplitude and phase, the reflection from a flat sediment surface will not produce a perfectly spherical reflection, but a distorted semishperical wave. Furthermore, if the sediment has a higher sound speed than the fluid above, there is a critical angle past which there is total internal reflection. At this critical angle, the acoustic wave propagates along the fluid/sediment interface at the speed of sound in the sediment and rereflects back into the fluid at the same critical angle. This phenomenon is called the head wave or lateral wave since it travels ahead of the reflected acoustic wave in the water. Figure 1 shows the various ray paths between a source and a receiver that are possible with spherical waves. The interested reader can find color pictorials of lateral waves in Ref. 9.

From the figure, the direct path (DP) is the shortest distance between a source and a receiver. The specular reflection path (RP) is incident on the sediment at a grazing angle dependent on the position of the source and receiver. The lateral wave path represents an additional ray that may cause interference with specular reflection. It is simplistic to consider only the ray theory approximation of the lateral wave when addressing spherical effects since, around the critical angle, ray theory will produce infinite intensity at caustics, points in space where rays cross. In order to calculate the spherical wave reflection coefficient, the evaluation of the entire field integral is necessary.\(^10\) Also, it is important to note that spherical wave effects will only be prevalent for source receiver ranges of less than 100–200 wavelengths and only for small grazing angles. For example, at 140 kHz in seawater, spherical effects are negligible after about 2 m, but at 40 kHz, spherical effects are prevalent up to about 5–7 m subcritically. This is due to attenuation and spreading of the lateral wave.

Section 26 in Brekhovskikh\(^1\) details the development of a method to represent a spherical wave as an infinite summation of plane waves. The plane wave reflection coefficients computed from various models can be linearly multiplied by the individual plane waves inside the integral. The resulting integration yields the spherical wave reflection coefficient. The first step in PWD is to assume the incoming wave has the form of a spherical wave in three dimensions where the time dependence \(e^{-i\omega t}\) has been suppressed and only relative pressures are considered

\[
P_i = \frac{P_0 e^{ikr}}{r},
\]

where

\[
r = \sqrt{x^2 + y^2 + z^2}.
\]
Brekhovskikh shows that by converting to polar coordinates and making use of several identities and substitutions the spherical wave reflection coefficient \( R_{SW} \) can be written in the form of

\[
P_t = \frac{P_0}{2\pi} \int_{-\infty}^{\infty} e^{i(k_x x + k_y y + k_z z)} \frac{dk_x dk_y}{k_z}.
\]

(15)

Here \( h \) is the height above bottom for the transducer array, \( \theta \) is the grazing angle, and \( P_t \) is the incident spherical wave. Equation (16) is the PWD integral that is used to modify the plane wave reflection models for comparison with spherical wave data. It yields the reflected pressure for a given geometry based on the height above the sediment and the range between the transducers. In order to modify a given plane wave reflection coefficient model, the plane wave reflection coefficient, \( R(f, \theta_i) \), computed from the model of interest is linearly multiplied by the kernel of the PWD integration.

Although traditional analysis involves many simplifications of Eq. (16) and typically various forms of saddle point integration, current computer processing power facilitates the direct numerical integration in this work. This greatly simplifies the conceptual procedure.

In the past, numerical integration has successfully been used to verify Brekhovskikh’s work.\(^{11}\) Until recently, however, the computational requirements were too high to make this approach a feasible means of solving repeated integrals.

Numerically integrating Eq. (16) near normal incidence and near zero degrees grazing angle is difficult. The frequency of oscillation of the function is much higher at normal incidence than at other angles. In this region, a sufficiently fine grid size results in large computation time. As the grazing angle decreases, the exponential decay in the imaginary portion of the PWD kernel decreases and a sinusoidal component becomes dominant. The limit occurs at exactly zero degrees where the imaginary portion is a varying sinusoid to minus infinity. However, with the transducer geometry used in this experiment, it was impractical to measure data at grazing angles of more than about 70° grazing or less than about 7° grazing. Therefore, it was unnecessary to numerically calculate the spherical wave reflection coefficient at the limits.

Figure 2 shows the comparison of the numerical integration of PWD integral with the plane wave reflection coefficient for a fluid model. The results were computed at two different separation distances and frequencies to show the general trends. As suspected, lower frequencies and closer distances cause greater spherical effects. Note that even at 1.27 m apart and at 112 kHz, there is some indication of spherical wave effects.

### III. EXPERIMENTAL SETUP

Reflection coefficient measurements were taken in the ARL:UT Sonar Modal Calibration Tank 1. The tank is 18.3 m long, 4.5 m wide, and 3.6 m deep. At the bottom there is 1 m of unwashed river sand that was collected from the lower Colorado River in the 1970s. It has remained generally undisturbed for over 30 years. Biological activity is kept to a minimum through periodic chlorination. The tank has a slight temperature gradient of 0.45 °C/m extending approximately 0.3 m below the air/water interface. Below 0.3 m depth, the temperature is constant. Situated above the tank are two moving platforms, each equipped with a vertical shaft capable of positioning 150 lb of equipment at any depth in the water column. It provides an ideal location to precisely measure the reflection coefficient of sediments as a function of grazing angle.

The sediment in the tank includes various inhomogeneities commonly found in river sand. Though it is generally fine sand, there are numerous rocks, and chunks of clay. A 2002 study characterizing the sediment found the sound speed to be 1740±20 m/s at 30 kHz.

The average total density was measured to be 1.99 g/cm³. A detailed grain analysis was performed in 1992. A visual analysis at that time revealed a majority of quartz crystals with some silt and feldspar. The majority of the grains were poorly sorted and angular to subangular in shape. The porosity was measured to be 0.40. The average grain size diameter is 0.25 mm based on sieve measurements.

Figure 3 shows an elevation schematic of Calibration Tank 1. The moving platform can horizontally traverse the entire length of the tank. Attached to the vertical column is a two transducer acoustic test frame. The frame itself is narrow in depth and can easily be reversed to take data both from the sediment/water interface or the air/water interface. The frame was engineered so that it had less than a 1 mm deflection in either configuration. For the sediment/water interface data collection at the larger range, a cork reflector was mounted at a 45 deg angle above the sending transducer to exclude the air/water surface reflection for the sediment data. (See Fig. 4.)

\[
R_{SW} = \frac{P_0 d k}{P_t} \int_{\pi/2-\infty}^{\pi/2+\infty} R(\theta) J_0(k r \sin \theta) e^{i 2 k h \cos \theta} \sin \theta d \theta.
\]

(16)
Each transducer was a spherical ITC 1089 transducer positioned so that the $XY$ planes of the transducers were aligned. The $z$ axis points along the transducer pigtail. The beam patterns of the two spherical transducers are nominally constant to 1 dB about the $XY$ plane.

The reflection coefficient as a function of angle was measured by changing the height of the transducer array to sample different angles. Figure 1 shows that a change in height above the interface directly corresponds to a change in the grazing angle, $\theta_g$, assuming the range, $r$, is held constant. The individual rays indicate the arrivals via the direct path and reflected path. The direct path will experience normal spherical spreading and will decrease in amplitude at a rate of $1/r$. The reflected path will experience a similar spreading loss due to the added distance, $\cos \theta_g/r$ where $\theta_g$ is the grazing angle.

To insure spatial stability, approximately 30 different locations on the sand were sampled. At each location, 75 angle measurements were made by coherently averaging 300–500 individual pings together. A 1 ms linear frequency modulated chirp, from 30 to 160 kHz, was used to sample a range of frequencies with each ping. Bandpass filtering allowed 10 subbands to be analyzed in each ping. This method uses one measurement to take the place of ten measurements at individual frequencies. The above procedure was conducted at two different ranges between the transducers to compare the spherical effects as a function of range. The collected data represent one of the most comprehensive acoustic analysis of the sediment in ARL:UTs calibration tank to date.

Rough interface scattering effects were minimized by smoothing the sand with a 4 in. PVC pipe filled with scrap steel. The weighted pipe was gently dragged across the bottom several times. The laser line scan method developed at ARL:UT could not detect any surface roughness. This method has millimeter accuracy for spatial wavelengths from 0.1 to 1 m.12

The average root-mean-square roughness of smoothed, dry sand taken from the calibration tank was measured to be approximately 0.5 mm. This is about twice the average grain size, but is not unexpected. Various effects can increase the apparent roughness such as imperfections in the smoothing process, adhesion between sand grains and grain size distribution. Roughly 62% of the grains by mass are larger than the average grain size diameter.* The packing order of the individual grain sizes may also affect the apparent roughness.

An experiment test frame was designed and built in order to provide a stable platform that accommodated both air/water interface and sediment/water interface measurements. The sediment reflection measurements were normalized by the air/water data in order to correct for beam pattern and transducer response effects. This method improves the accuracy of the transducer response to nominally 0.25 dB.

The entire experiment, broken down into transfer functions, is shown in Fig. 5. The analog to digital (ADC) and digital to analog (DAC) converters on a National Instruments™ PCI-6115 data acquisition card were configured to trigger off of the same 4 MHz clock. A linearly amplified

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signal from the DAC is applied directly to the ITC 1089 transmit transducer, \( T_s \). This produces relatively low amplitude, frequency dependent sound pressure levels of 155–170 dB in the water column. Because the transducers deviated from an ideal spherical source, slight beam pattern effects, \( B \), contributed to the overall signal. The received signal also includes effects from spherical spreading, \( S_p \), and the spherical reflection from the interface, \( R(f, \theta_g) \). The received signal \( (X_r) \), including both the direct path pressure \( (X_{DP}) \) and reflected pressure \( (X_{RP}) \), is modified by the receiving transducer response including amplification.

IV. SIGNAL PROCESSING

Measuring the spherical wave reflection coefficient required accounting for several additional effects, in addition to the spherical wave effects, that were inherent in the experiment.

Each of the physical steps in the experiment, as outlined in Sec. III, can be reduced to transfer functions for signal processing. These transfer functions operate on the Fourier transform of the pressure amplitude

\[
\eta_{DP}(f, \theta_b) = |T(f)B(f, \theta_b)S_p(\theta_b)|, 
\]

\[
\eta_{RP}(f, \theta_g) = |T(f)B(f, \theta_g)S_p(\theta_g)R(f, \theta_g)|. 
\]

In general, two signals can be isolated in the data, the direct path pressure amplitude and the received path pressure amplitude. The amplitude of the direct path \( (\eta_{DP}) \) includes the transducer response \( TR \), the beam pattern \( B \), and spreading \( S_p \) effects the direct path angle \( \theta_b \) which was zero degrees grazing. The transducer response includes all effects from the amplifiers and DA/AD conversions. The beam pattern effects include both source and receiver beam patterns. The amplitude of the reflected path \( (\eta_{RP}) \) includes the same effects as the direct path, but are dependent on the particular grazing angle of each measurement. In addition, the reflected path includes the reflection \( R \) effects associated with interaction with the sediment or the air/water interface. These simplifications allow the different received pressure amplitudes to be described as

\[
X(f) = I(f) + N(f), 
\]

\[
X_{DP}(f, \theta_b) = X(f) \eta_{DP}(f, \theta_b) + N_{DP}(f), 
\]

\[
X_{RP}(f, \theta_g) = X(f) \eta_{RP}(f, \theta_g) + N_{RP}(f). 
\]

The input signal \( X \) includes only the initial wave form \( I \) with some small added electronic random noise \( N(f) \). Both the direct path amplitude \( X_{DP} \) shown in Eq. (20) and the reflected path amplitude \( X_{RP} \) shown in Eq. (21) include the input signal, but have the additional effects associated with each. The respective noise terms in each equation represent the contribution of the noise in Eq. (19) as well as added random noise \( N(f) \) associated with each path. For the direct path the noise is associated with local temperature variations in the water column and is expected to be small. Some possible noise contributions for the reflected path are discussed in Sec. V E. The sum of the direct and reflected paths was recorded. However, since the distance between the transducers is constant, the direct path contribution for any measurement is the same. Therefore, the direct and reflected paths can be separated by subtracting a temporally isolated direct path that was recorded in the middle of the water column before each set of measurements.

The process for calculating the reflection coefficient from a measured time series is as follows. First, the direct and reflected path arrival times were determined using a time domain correlation with a calculated replica based on the input signal. Next, the grazing angle was determined from the delay between the direct path arrival and reflected path arrival. A temporally isolated direct path signal was subtracted from the recorded time series leaving only the reflected path signal and associated noise. Then, a second correlation was performed using a temporally isolated direct path signal as the replica. The signal was then bandpass filtered on the data to isolate each frequency band. The initial specular bistatic amplitude and phase were determined by comparing the reflected path amplitude and phase at the peak of the correlation to the direct path amplitude and phase. Last, the data were corrected for transducer response, beam pattern, and spherical spreading by normalizing with the air/water interface response determined in the same manner.

A. Replica correlation and time compression

For this work, the replica correlation and associated time compression were accomplished using a Wiener filter based on the expected signal, \( x_{DP} \).

\[
F_{RP,DP}(f, \theta_g) = \frac{X_{DP}(f, \theta_b)}{|X_{DP}(f, \theta_b)|^2}X_{RP,DP}(f, \theta_g)(BP(f)). 
\]

Here \( F_{RP,DP} \) is the Fourier transform of the filtered signal. \( X_{DP} \) is the Fourier transform of the expected signal. \( X_{RP,DP} \) is the Fourier transform of the raw data and \( BP(f) \) is the pass band. \( e \) is the noise floor.

A sample Wiener filtered time series is shown in Fig. 6. The Wiener filter is constructed so that the time delay of the direct and reflected paths are absolute.
Frequency resolution is established by band pass filtering the direct and reflected paths. The pass bands were spaced every 12 kHz. Chebyshhev windows in the frequency domain were used to enhance the time compressed peaks and suppress side lobes in the time domain.

The disadvantage to using the Wiener filter method is the limited frequency resolution. The peak width is inversely proportional to the bandwidth of the signal. If the bandwidth is increased, then the peak width in the time domain will increase. If the bandwidth is too small, the time series width becomes too large to distinguish the reflected path arrival from scattered arrivals from the walls of the tank. The bandwidth of the band pass filters was optimized so that the direct and reflected peaks were resolvable.

B. Determining the grazing angle

The grazing angle is calculated by taking the arc-cosine of the direct path length over the reflected path length. Since the sound speed in the water column is relatively constant, distance and time are proportional. Therefore, taking the ratio of the direct and reflected path length is equivalent to taking the ratio of the direct and reflected arrival times, $t_{DP}$ and $t_{RP}$, respectively.

$$\theta_g = \cos^{-1} \left( \frac{t_{DP}}{t_{RP}} \right). \quad (23)$$

Figure 6 shows how Wiener filtering over the entire frequency range (30–160 kHz) was used to precisely locate the arrival times of the direct and reflected paths. The figure also illustrates how the change in height above the bottom increases the grazing angle and how the reflection coefficient ($R_g$) may be determined by the ratio of the peaks height of the correlated signal. In this figure, the initial reflection coefficient for 23.5° is −2 dB and for 45.5° it is −11 dB.

For the determination of the grazing angle, the expected signal used in the Wiener filter was the input signal modified by the amplitude of the direct path. Since the DAC and ADC clocks are synchronized, the transmitted input signal starts simultaneously with the actual recorded measurements. This absolute reference was critical to accurately determining the proper arrival time.

C. Determining the reflection coefficient

The initial reflection coefficient $[R(f, \theta_g)]$ is determined by the ratio of the peak of the reflected path correlation and the direct path correlation. This has the added benefit of accounting for transducer response.

The beam pattern and spreading losses were accounted for by normalizing with the data taken from the air/water interface. Assuming that the noise is negligible, the ratio of the reflection coefficient measurement from the air/water interface $\eta_a$ and that of the water/sediment interface $\eta_s$ is

$$\frac{\eta_t}{\eta_a} = \frac{B(f, \theta_g) S_p(\theta_g) R_s(f, \theta_g)}{B(f, \theta_g) S_p(\theta_g) R_a(f, \theta_g)} \cdot \quad (24)$$

Since the complex reflection coefficient for the air/water $R_a$ interface is known, normalizing the sediment initial reflection coefficient by the air/water initial reflection coefficient removes both the frequency and angular dependent beam pattern and the spherical spreading. For normalization, all initial reflection coefficients are first interpolated to a standard angle and frequency set. Then both the air/water interface and sediment/water interface data are averaged over all spatial locations.

D. Phase determination and normalization

For the phase measurement, the Hilbert transform of the time series was taken prior to filtering. This created a complex time series. The phase angle was determined at the peak of the correlation with the Wiener filter in the time domain. The absolute phase was determined by subtracting the phase of the direct path from the reflected path. As with the magnitude, the phase was corrected for beam pattern effects by normalizing by the air/water interface data.

V. RESULTS

A. Normalization measurements

Ten normalization measurements were taken over the entire angle range at each range by inverting the experimental frame and measuring the specular bistatic response from the air/water interface. These data were averaged and used to verify that the grazing angle calculations were valid and to remove beam pattern effects from the sediment measurements. Figure 7 shows the phase and magnitude for the 1.27 m data for the air/water interface at 54 kHz uncorrected for spreading losses. The data were averaged over nine sets. The dashed lines represent the 95% confidence bounds. The overall trend of the data matches well with theoretical spherical spreading (1/r). This indicates that the algorithms used to calculate the grazing angle are reliable. The variations can be attributed to the beam pattern. It is evident that the effects of
the beam pattern are small compared to the effects of the spreading. The beam pattern exhibits 0.1 dB oscillations especially in the 25–30 angle range at 54 kHz as evident in Fig. 7. These oscillations are not apparent at larger frequencies as seen in Fig. 8.

The frequency dependent beam pattern of the two phones was calculated by subtracting the theoretical spreading from the measured air/water interface reflection data. For the phase, the beam pattern effects is determined by subtracting 180° from the phase. The 1.27 m data set has slight variations of approximately ±1 dB for a 95% confidence interval over the entire frequency range. The phase difference can be as much as 50° depending on the frequency and angle. The error is generally around 10° when averaged over the broadband. The 0.25 m data set has similar characteristics up to approximately 80 kHz. At the higher frequencies, the 0.25 m data becomes unstable. This is shown in Fig. 9 for 125 kHz. The 0.25 m data may be more heavily influenced by volume properties due to a higher influence of the sediment born lateral wave and steeper ray angles which may penetrate further into the sediment. Therefore, only the 1.27 m data will be considered quantitatively.

B. Spherical wave effects

The normalized reflection coefficient results for the 1.27 m case and the 0.25 m case are shown in Figs. 10 and 11 for 54 kHz band with a 95% confidence interval. These are spatial averages of 30 1.27 m measurements and 35 0.25 m measurements along the sediment floor in the calibration tank. Although the critical angle should be located at approximately 30°, both data sets are heavily influenced by spherical wave effects which shifts the critical angle value. This inward shift of the critical angle, especially at lower frequencies, as well as the interference pattern in the subcritical region of the 1.27 m data are indicative of lateral wave interference.10

To demonstrate these effects, the plane wave elastic reflection coefficient and the spherical wave elastic reflection coefficient are calculated using plane wave decomposition and the visco-elastic model and plotted with the data (see Figs. 10 and 11). It is clear that predicting the data with only the plane wave model would lead to large errors especially in the critical region. The plane wave model overpredicts the location of the critical angle by 5° for the 1.27 m data and 12° for the 0.25 m data. The spherical wave model correctly
predicts the decrease in critical angle as well as the presence of an interference pattern subcritically. Additionally, the spherical wave model correctly predicts the dependence of spherical wave effects on transducer spacing although the 0.25 m data is higher than either model as the grazing angle approaches normal. The spherical wave effect is not present at angles larger than critical. The change in phase due to spherical wave effects is small and is not measured in this experiment. For the 1.27 m data both the plane wave model and spherical wave model correctly predict the phase. For the 0.25 m data, the phase is not predicted by either model in the critical region.

In all other model comparisons in this work, only the spherical wave reflection coefficients as calculated using plane wave decomposition will be considered. As stated above for the air/water data, the 0.25 m data were found to be unstable and will not considered quantitatively.

C. Calculation of the models for data comparison

A standard set of parameters were used to calculate all of the reflection coefficients based on the three models described above. The parameters were determined based on sand measurements made in the calibration tank and similar sand collected in the SAX’99 experiment, inversion results of the reflection data collected in the calibration tank, and values found in common reference tables. The attenuation in the water column is assumed to be zero. No attempts were made to adjust the parameters to reflect experimental results.

Table I shows the parameters used in the fluid and viscoelastic models. The fluid density was found in common reference tables. The fluid sound speed, measured as part of this study, was 1476±7 m/s. This value corroborates well with common reference tables. The average wet sediment density was calculated based on porosity and the density of the sand grains and water. The sediment sound speed in the calibration tank was measured and inversion results verified this result. The value for the compressional attenuation was taken from the SAX’99 measurements since the sediments at that site and that of the calibration tank are similar in composition and size distribution. The shear speed was obtained by inversion results and corroborated with measurements at 312 Hz. The shear attenuation, which will be shown to have little effect on the data, was extrapolated from measurements on unsorted sand at lower frequencies.

Table II shows the poroelastic parameters used in the grain shearing model and the effective density fluid model. The frame shear and bulk moduli are assumed to be zero as described in Sec. II. The porosity was obtained by inversion and is consistent with, values measured in the tank and recorded at SAX’99. The fluid bulk modulus was also determined by inversion and substantiated by common reference tables. Viscosity was determined from common reference tables. The grain density was based on a grain analysis in the tank which revealed that the sand consisted of 85% quartz and 15% feldspar. Grain bulk modulus was obtained by inversion of this data set and is in the 95% confidence interval measured for similar sediment. The value for tortuosity was taken from the SAX’99 measurements although the same value was determined by inversion. The pore size parameter was calculated using the Kozeny-Carmen equation and the permeability was assumed to be similar to the SAX’99 valued.

Table III shows the empirical values used in Buckingham’s model. The frequency at which the sediment compressions...
ional sound speed in the calibration tank was measured was 30 kHz. The frequency of the shear sound speed measurement is 312 kHz. The remaining values are from Table II. The plane wave reflection coefficient (PWRC) models calculated with the common parameters were modified with PWD to include spherical wave effects and then compared to the measured spherical wave reflection coefficient.

D. Data/model comparison

In order to compare the data with the predictive models, three frequency bands were chosen of the ten measured. These bands were centered at 54, 102, and 125 kHz. These data were indicative of the quality of the entire data set. The model predictions compared with the data are shown in Figs. 12–14. The data are shown with a 95% confidence interval.

Qualitatively, at 54 kHz, the elastic and grain shearing models are all underpredicting the bottom loss by almost 1 dB from the mean although these models do fit within the error. The EDFM, however, overpredicts the losses above the critical angle. However, this model is also within the 95% confidence interval. All three models predict the critical angle within about 3° and correctly predict the phase behavior.

For the higher frequencies, the elastic and grain shearing models under predict the value of the reflection coefficient at high grazing angles while the EDFM model predicts the mean value. In fact, both the fluid and grain shearing models predict values that are not within the 95% confidence interval of the data. EDFM model predicts a lower than measured

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TABLE III. The grain shearing model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp. frequency</td>
<td>30 kHz</td>
<td>Measured</td>
</tr>
<tr>
<td>Comp. sound speed</td>
<td>1740 m/s</td>
<td>Measured</td>
</tr>
<tr>
<td>Comp. attenuation</td>
<td>7.5 dB/m</td>
<td>SAX'99 dataa</td>
</tr>
<tr>
<td>Shear frequency</td>
<td>312 Hz</td>
<td>Measuredb</td>
</tr>
<tr>
<td>Shear sound speed</td>
<td>80 m/s</td>
<td>Measuredb</td>
</tr>
<tr>
<td>Shear attenuation</td>
<td>0.9 dB/m</td>
<td>Measured</td>
</tr>
</tbody>
</table>

aSee Ref. 16.
bSee Ref. 17.
value for of the reflection coefficient at the critical angle, however, the prediction is still within the 95% confidence interval. All three models correctly predict the phase behavior at 54 kHz although the phase measurement becomes more unstable at higher frequencies. At 125 kHz none of the models predict the phase behavior subcritically.

The three models were quantitatively compared by taking the mean absolute relative error averaged over the angle range at 54, 102, and 125 kHz. The mean relative error was also considered averaged over frequency at three angles, 15°, 30°, and 45°. The results are shown in Table IV.

The elastic model performs slightly better at low angles, while the poroelastic model is significantly more accurate at higher angles and high frequencies. This is in agreement to the results from Williams who found that at lower frequencies, the water is locked to the frame and the total mass density is in closer agreement with the effective density.3

A sensitivity analysis was performed on the EDFM model for three parameters: the fluid bulk modulus, the grain bulk modulus, and the tortuosity. The analysis was performed since the fluid bulk modulus and the grain bulk modulus were determined from an inversion of the same data set and the tortuosity was measured within a range at SAX 99,16. The range of parameters tested are given in Table V. The fluid bulk modulus range is determined to be a proportional range as determined at SAX 99 for seawater. The range of the grain bulk modulus is taken from the 95% confidence interval of the measurements in Ref. 19. The tortuosity range is given by the measurement at SAX 99.16 All possible combinations of these parameters were calculated and the range of solutions is shown in Fig. 15 compared with the measured data and the elastic solution for all angles at 125 kHz [Fig. 15(a)] and for all frequencies at an angle of 50 deg. [Fig. 15(b)]. As shown in the figure, the poroelastic model at all range combinations is significantly different from the elastic model at high grazing angles and a better fit for the data at high grazing angles and frequencies.

E. Data variance

The standard deviation at high grazing angles and high frequencies in the data is typically 2 dB or more. The variation of the data as measured with the scintillation index is highly dependent on both frequency and angle as shown in Fig. 16.

There are several effects that may contribute to the variation. These include interface scattering, volume scattering, layering, and gradients and variation of bulk sediment properties. Additional sources of error associated with the experimental technique or data analysis are not suspected

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_f$ Gpa</td>
<td>2.14</td>
<td>2.18</td>
</tr>
<tr>
<td>$K_r$ Gpa</td>
<td>32</td>
<td>49</td>
</tr>
<tr>
<td>$\tau$ m$^2$</td>
<td>1.19</td>
<td>1.57</td>
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</tbody>
</table>

FIG. 15. Sensitivity analysis of three parameters for the poroelastic model. Shown in the figure are all of the combinations of the parameter ranges from Table V compared to the measured data and the elastic model predictions. In (a) are shown the angle dependent measurements and predictions at 125 kHz. In (b) are shown the frequency dependent predictions at 50 deg. There is still significant difference between the poroelastic model predictions and the elastic model at high frequencies and grazing angles.

FIG. 16. Scintillation index of the data at 20 and 50 deg grazing as a function of frequency. The variability has a strong dependence on the frequency and grazing angle.
bulk variations as defined by the fluid or viscoelastic models are not suspected to account for the high degree of variation in the data. In fact, if these variations were due to spatially dependent bulk sediment properties, there would be roughly a 230% difference in the local sediment impedances along the tested area. This equates to sound speed and density changes on the order of 1200–3200 m/s and 0.9–5.4 g/cm³, respectively. These limits are significantly outside the realm of independent measurements.

The analysis of the effects which are causing the high variation in the data is outside the scope of this paper and will be considered in subsequent publications.

VI. CONCLUSION

PWD was used to modify three plane wave reflection coefficient models to include spherical wave effects. The resulting spherical wave reflection coefficients were compared to experimental measurements. The measurements were taken from unwashed river sediment from 5° to 75° grazing at frequencies from 30 to 160 kHz. Two different transducer ranges were used, 1.27 and 0.25 m. Thirty different locations were tested to insure statistical significance.

The data were shown to have significant contributions from spherical wave effects that were dependent on frequency and range. The plane wave models overpredicted the location of the critical angle by 5° and 12° for the 1.27 and 0.25 m data, respectively. The spherical wave model as calculated by plane wave decomposition correctly predicted the location of the critical angle. Plane wave decomposition did not significantly alter the phase prediction.

The 1.27 m data were compared with three current models for acoustic interaction with sandy sediments. Each model was calculated using plane wave decomposition to correct for spherical wave effects. It was found that the viscoelastic model and grain shearing model predictions were not within the 95% confidence interval at high frequencies and angles.

The data exhibited high variance which was frequency and angle dependent. This variance is not likely to be caused by variations in bulk properties as defined by the fluid or viscoelastic models. The cause of the variance will be considered in subsequent publications.

In conclusion, these reflection coefficient measurements suggest that plane wave decomposition is a viable method for computing the effects of spherical waves. Furthermore, the data suggest that the poroelastic model is the best predictor for the reflection coefficient from sandy sediments.

ACKNOWLEDGMENTS

The authors would like to thank the Office of Naval Research and CDR Robert Headrick for sponsoring this work. The authors wish to acknowledge the help of Dr. Gaetano Canepa at the NATO Underseas Research Center for much help with running the BoRIS simulations and Dr. Charles Holland at the Applied Research Laboratory, Penn State, for his many insightful comments. Last, thanks to Dr. Nicholas Chotiros for many helpful discussions.