Acoustic Source

• Air gun (5 in³ [also available 20 in³]):
  Source Level 165 to 175 dB/micro Pa
  Calibration Hydrophone @ 1m
  Frequency band 30 Hz to 500 Hz

Advantages:
  a) Very sharp pulse, provide a very good source for modal analysis
  b) Compact, mobile, easy to use
  c) Available and calibrated already

Disadvantage:
  a) Need to apply for permit “soon” in advance to the experiment.
Tripod Acoustic Systems

- **Tripod VLA:**
  - Eight receiving elements
  - 80 kHz sampling frequency
  - ~50 hours of underwater lifetime

- **Single Source**
  - ITC3013 source near the top (~4 m from the sea floor)
Tripod operations during KAM08
Geo-acoustic parameter estimation using a multi-step inversion technique based on normal mode method

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Known: water SSP, density, WD, SD, RD

Broadband acoustic signals

- Normal mode code
- Time Frequency Analysis
  - Calculated modal arrival time $T_n(f, c_b)$
  - Measured modal arrival time $T_n(f)$

Cost Function: $F_1(c_b, f) = \sum_n \left( \frac{f_n^2(f)}{c_b} - T_n(f, c_b) \right)^2$ (1)

$F_1$ is minimized by an optimization algorithm to obtain $c_b$ as a function of frequency

Inverted $c_b$

Method 1

Cross Spectral Density Matrix
- Singular Value Decomposition
- Data-derived mode depth function $\Phi(z_i)$

Cost Function: $F_2(k_n) = \sum_i \left( \Phi(z_i) - \Psi(z_i, k_n) \right)^2$ (2)

$F_2$ is minimized by an optimization algorithm to obtain $k_n$

$\Psi(z_i+1) = \Psi(z_i-1) + \left\{ 2 - h^2 \left( \omega^2/c^2(z_i) - k_n^2 \right) \right\} \Psi(z_i)$ (3)

Depth-separated finite difference wave equation

The impedance from the mode function

$Z_w = -\rho \frac{\partial \Psi}{\partial z}$ (4)

The impedance from bottom matching

$Z_b = -\rho \frac{\sqrt{k_n^2 - (\omega/c_b)^2}}{k_n^2 - (\omega/c_b)^2}$ (5)

Method 2

Step 1

Calculated modal attenuation coefficient

$\beta_n(\alpha_b) = \frac{\omega}{k_n} \int_{z_a}^{z_b} W(z) \rho \Phi(z) dz$

$\beta_n(\alpha_b, f) = \frac{\omega}{k_n} \int_{z_a}^{z_b} W(z) \rho \Phi(z) dz$ (6)

Cost Function: $F_3(\alpha_b, f) = \sum_n \left( \beta_n(\alpha_b) - \beta_n(\alpha_b, f) \right)^2$ (7)

$\phantom{1}$ Method 1

$\phantom{1}$ Inverted $\alpha_b$

Calculated modal amplitude ratios

$R_{n1}(\alpha_b) = \frac{\sum_n \left| \frac{\Psi_n(z)}{\Phi_n(z)} \right|^2}{\sum_n |\Psi_n(z)|^2 |\Phi_n(z)|^2} \left( 2 - \beta_n(\alpha_b) \right) \exp(-2\beta_n r)$ (8)

Method 2

Inverted $\alpha_b$

Cost Function: $F_4(\alpha_b, f) = \sum_n \left( R_{n1}(\alpha_b) - R_{n1}(\alpha_b, f) \right)^2$ (9)

Step 2

Calculated vertical coherence

$\gamma_{\nu\nu}(\alpha_b) = \sum_n \frac{|\Psi_n(z)|^2 |\Phi_n(z)|^2}{\sum_n |\Psi_n(z)|^2 |\Phi_n(z)|^2} \left( 2 - \beta_n(\alpha_b) \right) \exp(-2\beta_n r)$

Method 3

Inverted $\alpha_b$

Cost Function: $F_5(\alpha_b, f) = \sum_n \left( \gamma_{\nu\nu}(\alpha_b) - \gamma_{\nu\nu}(\alpha_b, f) \right)^2$ (10)

Calculated longitudinal horizontal coherence

$\gamma_{\mu\mu}(\alpha_b) = \sum_n \frac{\left| \frac{\Psi_n(z)}{\Phi_n(z)} \right|^2}{\sum_n |\Psi_n(z)|^2 |\Phi_n(z)|^2} \left( 2 - \beta_n(\alpha_b) \right) \exp(-2\beta_n r) \exp(-ik_n DL)$

Method 4

Inverted $\alpha_b$

Cost Function: $F_6(\alpha_b, f) = \sum_n \left( \gamma_{\mu\mu}(\alpha_b) - \gamma_{\mu\mu}(\alpha_b, f) \right)^2$ (11)
Experimental Description
Experimental Description
Data-model comparison of modal dispersion curves

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![Graph showing data-model comparison of modal dispersion curves](image)
Seabed attenuation inversion using LHC

The experimental longitudinal horizontal coherence

\[ \rho(\Delta T, \tau) = \frac{\int_{t}^{t+\Delta T} p_1(t)p_2(t-\tau)dt}{\sqrt{\int_{t}^{t+\Delta T} p_1^2(t)dt \times \int_{t}^{t+\Delta T} p_2^2(t)dt}} \]

Where \( \tau = L/c \) is the time delay and \( \Delta T \) is the integration time.
LHC as a function of hydrophone separation-wave length ratio

![Graph showing LHC as a function of hydrophone separation-wave length ratio. The graph plots correlation coefficient against hydrophone separation-wave length ratio for different frequencies (100Hz, 200Hz, 400Hz). The data points decrease as the hydrophone separation-wave length ratio increases.]
Seabed sound attenuation inversion using LHC

Using the normal mode expression for sound pressure generated by a harmonic point source, we obtain the mathematical expression in terms of normal modes for the theoretical normalized longitudinal horizontal coherence:

\[ \gamma_{HL}(z_s, z, \Delta L, r) = \frac{\sum_n |\Psi_n(z_s)|^2 |\Psi_n(z)|^2 k_n \exp(-2\beta_n r) \times \exp(-ik_n \Delta L)}{\sum_n |\Psi_n(z_s)|^2 |\Psi_n(z)|^2 k_n \exp(-2\beta_n r)} \]  

(2)

where, \( r \) is range, \( z_s \) is source depth, \( z \) is receiver depth, \( \Delta L \) is the horizontal separations of the pair of hydrophones, \( \Psi_n \) is the mode depth function of the nth mode, \( k_n \) is the horizontal wave number of the nth mode, \( \beta_n \) is the modal attenuation coefficient, and \( S_n \) is the cycle distance of the nth mode. The square-average depth function can be calculated:

\[ \overline{\Psi_n(z)^2} = \frac{1}{S_n \sqrt{k^2(z) F(z) + k^2(z) - k_n^2}} \]  

(3)

\[ F(z) = 0.875 \left( \frac{1}{\pi f} \frac{dc(z)}{dz} \right)^{2/3} \]  

(4)

Inverted bottom attenuation in SW06 area

\[ \alpha_b = 0.3 f^{1.6} \text{ (dB/m)} \]